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better kind is \$1 and of the inferior \$.75 per lb. Now, he takes 9 parts of the better and mixes it with 2 parts of the inferior, then 9 parts of the mixture with 2 parts of the inferior, etc. What is the price of the  $n$ th mixture per lb.?

22. Proposed by F. P. MATZ, M. A., M. Sc., Ph. D., Editor of the Department of Mathematics in the "New England Journal of Education" and Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

For the sum  $D=$ \$30, Messrs. Zerr and Ellwood contracted to plough the sod for a circular track, width  $m=$ 60 feet and inner radius  $r=$ 940 feet. How is the money to be divided, if they commence ploughing at the inner circumference of the track, make uniform furrows of width  $n=$ 1 $\frac{1}{2}$  feet, and Mr. Ellwood continually follows Mr. Zerr during the ploughing?

Solutions to these problems should be received on or before July 1st.



## GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

### SOLUTIONS TO PROBLEMS.

6. Proposed by EARL D. WEST, West Middleburg, Logan County, Ohio.

Having the sides 6, 4, 5, and 3 respectively of a trapezium, inscribed in a circle, to find the diameter of the circle.

II. Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia; and J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio; and CHARLES E. MYERS, Canton, Ohio.

Let  $ABCD$  be the quadrilateral inscribed in a circle,  $AC$  the diagonal,  $AB=a=5$ ,  $BC=b=3$ ,  $DC=c=6$ , and  $DA=d=4$ .

From the triangles  $ABC$  and  $CDA$

$$AC^2 = a^2 + b^2 - 2ab \cos B$$

$$AC^2 = c^2 + d^2 - 2cd \cos D = c^2 + d^2 + 2cd \cos B$$

$$\therefore \cos B = \frac{a^2 + b^2 - c^2 - d^2}{2(ab + cd)}.$$

$$\sin^2 B = 1 - \frac{(a^2 + b^2 - c^2 - d^2)^2}{4(ab + cd)^2}$$

$$= \frac{(c+b+d-a)(a+b+d-c)(a+c+d-b)(a+b+c-d)}{4(ab+cd)^2}$$

$$= \frac{16(s-a)(s-b)(s-c)(s-d)}{4(ab+cd)^2} \quad \text{where } s = \frac{1}{2}(a+b+c+d)$$

$$AC^2 = c^2 + d^2 + \frac{2cd(a^2 + b^2 - c^2 - d^2)}{2(ab + cd)} = \frac{(ac + bd)(ad + bc)}{(ab + cd)}.$$

$$R = \text{radius} = \frac{AC}{2 \sin B} = \frac{1}{4} \sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(s-a)(s-b)(s-c)(s-d)}}$$

$$2R = D = \frac{1}{2} \sqrt{\frac{(39)(42)(38)}{(4)(6)(3)(5)}} = \frac{1}{2} \sqrt{172.9} = 6.57457225.$$

Also solved by *P. S. BERG, H. C. WHITAKER, J. R. BALDWIN, P. H. PHILBRICK, J. W. SCHEFFER.*

[Note.—The formula for the area of an inscriptable quadrilateral is  $A = \sqrt{s(s-a)(s-b)(s-c)(s-d)}$ , where  $s = \frac{1}{2}(a+b+c+d)$ .—Ed.]

7. Proposed by **WILLIAM HOOVER**, A. M., Ph. D., Professor of Mathematics and Astronomy in the Ohio University, Athens, Ohio.

Through each point of the straight line  $x = my + h$  is drawn a chord of the parabola  $y^2 = 4ax$ , which is bisected in the point. Prove that this chord touches the parabola  $(y + 2am)^2 = 8a(x - h)$ .

**II. Solution by L. E. PRATT, Tecumseh, Nebraska.**

Let  $y = Bx + C \dots (1)$  be a straight line cutting the given parabola. The co-ordinates of the middle point of the chord intercepted by the curve are

$$\left( \frac{2a - BC}{B^2}, \frac{2a}{B} \right).$$

Substituting these for  $x$  and  $y$  in the equations of the given straight line, we have

$$BC = 2a - 2amB - hB^2 \dots (2).$$

If  $(x_1, y_1)$  be any point of (1) we have  $y_1 = Bx_1 + C \dots (3)$ .

Eliminating  $C$  from (2) and (3) we obtain a quadratic in  $B$  which may be written

$$B = \frac{y_1 + 2am \pm \sqrt{(y_1 + 2am)^2 - 8a(x_1 - h)}}{2(x_1 - h)} \dots (4).$$

This result shows that two chords bisected by the straight line  $x = my + h$  may be drawn through the point  $x_1, y_1$ ; that when the roots of  $B$ , the angular coefficient, are equal the two chords coincide in one; that this takes place when the radical in (4) is equal to zero. But when the radical equals zero the point  $x_1, y_1$ , is a point of the parabola

$$(y + 2am)^2 = 8a(x - h)$$

and the straight line (1) is tangent to it.

This problem was solved in a very excellent manner by *Professors HUME, SCHEFFER, and ZERR*.

9. Proposed by **J. C. GREGG**, Superintendent of Schools, Brazil, Indiana.

Two circles intersect in  $A$  and  $B$ . Through  $A$  two lines  $CAE$  and  $DAF$  are drawn, each passing through a centre and terminated by the circumference. Show that  $CA \times AE = DA \times AF$ . [Euclid.]

**Solution by Miss GRACE H. GRIDLEY, Student in Kidder Institute, Kidder, Missouri.**

Let the straight lines  $CAE$  and  $DAF$  pass through the point of intersection  $A$  of the two circles, and through the centers  $O$  and  $O'$ , respectively.